Rec'd Dec. 27.　　　24 December 33　　1

Ansrd ″ 30.[1]

Dear Mr. Silberstein,

I ask you to please be so kind as to pardon my mistake. It is thus true that there is a static solution with only two point singularities. What does this mean for the general theory?

First of all, it is clear that the general foundation of the theory in the following sense involves the correct law of motion. The equivalence principle shows that when a mass describes a straight line (four-dimensionally) in field-free space in relation to a coordinate system for which the $g_{\mu\nu}$ are constant, something that is generally to be expressed in covariant form as movement in a <u>geodesic</u> line, this must also be the law of motion in the general field (geodesic line), in the event that only the <u>first</u> derivatives of the $g_{\mu\nu}$ of the <u>external</u> field enter into the law of motion. —

This is naturally not sufficient to provide a foundation for the law of motion. First, it is questionable whether the theory supplies the law of inertia for the field-free space; second, it is questionable whether only[2] the first derivatives of the $g_{\mu\nu}$ of the external field (this last concept is difficult to grasp precisely) enter into the law of motion.

A strong theory would undoubtedly have to proceed as follows: In a pure gravitational field there are no masses. Singularities are to be excluded in a field theory as a matter of principle, for when

[1] Translator's note: These two notations are in English in the original.

[2] Translator's note: "Only" is added above the line.

the kind of singularity is not specially determined (something that would be arbitrary and ugly in a general theory), the singularity signifies a place of lawlessness or arbitrary boundary conditions on the surface of a world-tube excluding the singularity.

In order to exclude singularities, the theory would have to be supplemented by means of those field variables that describe matter and among which the energy tensor T_{ik} would have to be expressed in relation to matter. The gravitational equations then have the form

$$R_{ik} - \frac{1}{2} g_{ik} R = -\kappa T_{ik} \ .$$

However, up to now no such complete theory, one that describes matter in a way free from singularities, has successfully been proposed.

Second, it is possible, while temporarily renouncing a physically deep theory of matter, to give a kind of phenomenological[3] (hydrodynamic) theory of matter while using the concepts of density, pressure, velocity. In this way it is possible to complete the theory at least formally. As is well known, this leads to the correct equations of motion. In order to see this last, the simplest method is to leave aside pressure as well and posit

$$T^{ik} = \rho u^i u^k \qquad \left(u^i = \frac{dx_i}{d\Delta}\right)$$

(dust-like matter). One thus immediately obtains the law of the geodesic line by constructing the divergence.

Namely, from the field equation follows $T^{ik}; k = 0$ or here specifically $\left(\rho u^i u^k\right); k = 0$ or $u^i; k u^k \rho + u^i (\rho u^k); k = 0$. The second term disappears (it multiplies the differential with u_i). Working out the first results in the geodesic line.[4]

---

[3] Translator's note: "Phenomenological" is added above the line.
[4] Translator's note: This paragraph ("Namely . . . geodesic line") is added in the bottom margin.

Third, however, one can also expect that the correct law of motion is obtained if one allows singularities, supposes the equation $R_{ik} = 0$, and takes care that the singularities have temporally constant and spatially[5] centrally symmetric character.

At this point we must now consider your solution. This is made more difficult to a certain degree by the fact that the choice of the spatial variables does not make it easy to recognize the centrally symmetric character.

First, it would have to be investigated whether the solution for <u>one</u> mass can also be put into a form on the basis of which their centrally symmetric character is recognizable. So far as I know, H. Weyl has done this.

In your solution, the character of the singularity in the area around m_1 is defined by

$$v = constant - \frac{M_1}{r_1}$$

$$\mu = constant - \frac{x_1^2}{2}\frac{M_1^2}{r_1^4} + 2\frac{M_1 M_2}{a^2}\left[\sqrt{1 - \frac{x_1^2}{r_1^2}} - 1\right]$$

Somewhat easier to see at a glance, supposing that $\frac{x_1}{r_1} = \cos\theta$,

$$\mu = constant - \frac{1}{2}\frac{M_1^2}{r_1^2}\cos^2\theta - 2\frac{M_1 M_2}{a^2}(1 - \sin\theta)$$

In any case, it appears that the second term, co-caused[6] by M_2, influences the character of the singularity and destroys the centrally symmetric character of the singularity.[7] However, this would have to be more closely investigated

[5] Translator's note: "Spatially" is added above the line.
[6] Translator's note: "Co-caused" is corrected above the line from "caused."
[7] Translator's note: "Of the singularity" is added above the line.

by introducing spatial polar coordinates in the area around m_1.

In any case, your investigation clearly shows how careful one must be in dealing with singularities, and how empty a field theory is that allows singularities without exactly determining their character.

Friendly greetings from your

A. Einstein