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Ansrd " 20. a.m. (˙/.)[1]

To
Professor Dr. Silberstein
129 Seneca Parkway
<u>Rochester, N.Y.</u>

Dear Mr. Silberstein,

I was at first very perplexed about your static example with the two masses, as I believed you that the space external to the mass points was regular. I was so much the more amazed as I had myself earlier showed that singularities already appear when calculating the second approximation.

In truth, however, the solution you give on the - axis is singular, as the following consideration shows. Your <u>spatial</u> line element is given by

$$d\sigma^2 = e^{2(\mu-\nu)}(dx_1^2 + dx_2^2) + e^{2\mu}x_1^2 dx_3^2$$

[figure]

We now take a plane E perpendicular to the x_2 axis, intersected by *the* x_2 axis at P.[2] In such a plane

$$d\sigma^2 = e^{2(\mu-\nu)}dx_1^2 + e^{2\mu}x_1^2 dx_3^2$$

On a straight line through P

$$d\sigma = e^{\mu-\nu}dx_1$$

On the circumference of a circle centered on P

$$d\sigma = e^{\mu}x_1 dx_3$$

Now I suppose that the circle has an (infinitely small) radius $(x_1 = R)$ and obtain as the ratio Z of the circumference to the radius (measured using measuring rods)

$$Z = \frac{\int_0^{2\pi} e^{\mu}x_1 dx_3}{\int_0^R e^{\mu-\nu}dx_1} = \frac{e^{\mu}2\pi R}{e^{\mu-\nu}R} = e^{-\nu}2\pi$$

However, for an infinitely small circle this ratio must everywhere equal 2π at the limit, something that is precisely in this instance not the case for the x_2 axis. The calculated field is therefore everywhere singular on the x_2 axis.

From this there follows first of all the invalidity of your example. More interesting would be the <u>proof of the nonexistence</u> of a static solution (whose singularities have simple polar character). I have shown this earlier at least for the second approximation (and that for a "correctly" accelerated mass the singularity disappears). It therefore does seem hardly possible to doubt that the

[1] Translator's note: These two lines are in English in the original.
[2] Translator's note: "Intersected by the x_2 axis at P" is added above the line, and "the" is handwritten.

field equations contain the law of motion, hence that the hypothesis of the geodesic line is superfluous.

Nevertheless, a truly complete theory would be at hand only if the "matter" in it could be represented in terms of fields and without singularities.

With the best of thanks for your friendly personal words as well, sincere greetings from

Your

A. Einstein

Copy of my reply (mailed 20 / XII. '33).[3]

129 Seneca Parkway

Rochester, N.Y.

December 20th, 1933.

Dear Professor Einstein,

I wish to thank y. for yr kind lett. of Dec. 17th. Your verdict, however, I am sorry to say, is quite wrong. You have inadvertently misplaced the two exponents ν *and* μ.

As in my first letter,

$$ds^2 = e^{2\nu}dx_4^2 - e^{-2\nu}\{e^{2\mu}(dx_1^2 + dx_2^2) + x_1^2 dx_3^2\}. \qquad (1)$$

Thus the circumf. of the circle you are contemplating is

$$C = 2\pi R e^{-\nu},$$

[figure]

and its radius

$$\rho = R e^{\mu-\nu},$$

whence

$$\underline{C/\rho = 2\pi e^{-\mu}} \qquad (\underline{\text{not}}\ e^{-\nu}2\pi).$$

Now,

$$\mu = -\frac{x_1^2}{2}\left(\frac{M_1^2}{r_1^4} + \frac{M_2^2}{r_2^4}\right) + \frac{2M_1M_2}{a^2}\left[\sqrt{1 - \frac{a^2x_1^2}{r_1^2r_2^2}} - 1\right] \qquad (2)$$

vanishes rigorously for $x_1 = R \to 0$, *so that*

$$\lim \frac{C}{\rho} = 2\pi.$$

Thus the solution (1), with (2) and $\nu = -\frac{M_1}{r_1} - \frac{M_2}{r_2}$, *satisfies also your own requirement of regularity (elementally Euclidean behaviour). The statements made in my first letter remain, therefore, in full vigour. Against your expectation, a statical solution with two (and, similarly, 3 or more) "singularities of simple polar character" does exist and, in view of its physical implications, it is imperative to deal with it in a fundamental*[4] *way in order to uphold your gravitation theory.*

I shall expect, with much interest, your views on this matter.

With cordial greetings, yours sincerely, L. Silberstein

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[3] Translator's note: All in English in the original.

[4] Translator's note: "Fundamental" is corrected from "fundamentally."