Berlin, 27 July 1901

Very esteemed Doctor,

I have read through your treatment of symbolic integrals of the electromagnetic equations with great interest, and even if I have not verified all the calculations, I have nonetheless become convinced that your essay sets out a series of thoughts that will perhaps prove to be very valuable for the future and is therefore suited for publication in the *Annalen der Physik* in any event.

Since you also ask me for details about my opinion and even offer to allow me to make changes in your manuscript, however, I am also happy to indicate to you the main point in which I consider that your paper could still be improved: that is, a somewhat more precise restriction or limitation of the theorems you formulate, whose scope can easily be overestimated according to your presentation.

Your entire theory is based on the assumption of the expandability of a function in a Taylor series,[1]

[1] Translator's note: "Series" is added below the line.

so for example,

$$E_t = E_0 + \frac{t}{1!}E_0' + \frac{t^2}{2!}E_0'' + \frac{t^3}{3!}E_0''' + \ldots$$

In order for this series expansion to be valid, however, it is not sufficient that each of the derivatives E_0', E_0'', E_0''', . . . be finite and continuous (at least for real t); so for example, the function $E_t = e^{-\frac{1}{t^2}}$ with all its derivatives is finite and continuous, and nevertheless, it cannot be put into the form of a Taylor series for $t = 0$. Of course, as I happily concede, those are singular cases. More important is the following: Quite correctly, you limit the sphere of validity of your generally formulated theorems to "epochs" within which the conditions for continuity are everywhere met. This limitation is more far-reaching than you seem to assume. You say (p. 6) that the difficulties of a discontinuity surface can be overcome by considering the discontinuity surface as a limit case of a so-called continuous transition layer. This is incorrect. <u>It is entirely impossible to select the transition layer in such a way</u>

that the Taylor series expansion is valid at all of its points. True, it is no doubt possible to select the transition layer in such a way that the function itself and its first, second, third . . . derivatives remain continuous at all of its points—and this is often put to advantageous use. But you need more for your theory, namely that it can be expressed by a Taylor series.

For example, let us take the simple case in which a function $f(x)$ is constant for a value of $x < x_0$ and also constant for a value of $x > x_0$, but of a different quantity. It is then entirely impossible to find a transition layer, however thick, such that $f(x)$ can be everywhere expanded in a Taylor series. That would mean representing a constant as the analytic continuation of another constant.

Connected to this is the fact that your formulas do not avoid "propagating" inconveniences originally limited to a certain narrow area. This is not due to the invalidity of these laws of propagation, but rather far more to the limited validity of your formulas.

Naturally, everything remains correct if you limit yourself to the epochs mentioned above.

Now, for obvious reasons, I would rather not intervene in your paper on my own authority, and therefore I prefer to send the manuscript back to you once more, leaving it to you to undertake a revision corresponding to my suggestion. After that, the best thing will be for you to send the manuscript directly to Prof. P. Drude[2] in Giessen, Nahrungsberg 8, and you may certainly mention me in doing so.

With the best of greetings, your devoted

M. Planck

[2] Translator's note: "Drude" is corrected from "Druden."